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Improving the Stability and Efficiency of the Implicit Pseudocompressibility Method

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I. Introduction

FOR a flow solver to be successfully applied in aerodynamic design, two of the most important objectives are obtaining a very stable solution and achieving a high efficiency. This is especially true for Navier–Stokes methods, which require very-fine-resolution grids, particularly for incompressible flows.

Many implicit schemes [1–11] have been developed and applied successfully to steady and unsteady flow simulations. Because of the high efficiency of the lower–upper symmetric Gauss–Seidel (LU-SGS) method [12], it has become very popular, and many improvements to this technique have been proposed in recent years. In [9], Yoon and Kwak developed an implicit LU-SGS method based on pseudocompressibility [13,14] for incompressible flows. Following this method, the LU factors are carefully constructed to make the lower and upper operators scalar diagonal matrices, which require only scalar diagonal inversions. The LU-SGS scheme, based on the pseudocompressibility method, is derived similarly to the implicit method for compressible flows.

Although the incompressible LU-SGS scheme is unconditionally stable in theory, it is proved by practices that stability and efficiency are still affected by viscous Jacobians, especially for low-Reynolds-number flows. According to the authors' experiences, as the Reynolds number of incompressible flows reduces, the stability of the scheme becomes worse and the efficiency will decrease if without viscous correction.

To improve the stability and efficiency, a viscous correction [3,8] on the LU-SGS scheme is performed in this paper. Based on incompressible Navier–Stokes equations with pseudocompressibility, the contribution of viscous Jacobians is considered.

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Several incompressible viscous flows are simulated by the present method to illustrate the effects of our improvement. The computed results are in good agreement with the experimental data and reference results. For simulations of low-Reynolds-number flows, the Courant–Friedrichs–Lewy (CFL) number can be increased greatly by stability analysis considering the contributions of the viscous Jacobians. Our results demonstrate that for low Reynolds number, with the contributions of the viscous Jacobians, the stability and efficiency of the LU-SGS method can be increased greatly compared with the method without viscous correction. It is very useful for the incompressible simulation of low-Reynolds-number flows.

II. Governing Equations and Implicit Schemes

With pseudocompressibility, the incompressible Navier–Stokes equations for a fixed control volume can be expressed in integral form as

$$\frac{\mathrm{d}}{\mathrm{d}t}\iint_V \boldsymbol{W}\,\mathrm{d}V + \int_{\partial V}\boldsymbol{F}\cdot\boldsymbol{n}\,\mathrm{d}S - \int_{\partial V}\boldsymbol{F}_v\cdot\boldsymbol{n}\,\mathrm{d}S = 0$$
$$\boldsymbol{W} = \begin{bmatrix} p \\ u \\ v \end{bmatrix} \qquad \boldsymbol{F} = \begin{bmatrix} \beta\boldsymbol{q} \\ u\boldsymbol{q} + p\boldsymbol{i}_x \\ v\boldsymbol{q} + p\boldsymbol{i}_y \end{bmatrix} \tag{1}$$
$$\boldsymbol{F}_v = \begin{bmatrix} 0 \\ \tau_{xx} \\ \tau_{yx} \end{bmatrix}\boldsymbol{i}_x + \begin{bmatrix} 0 \\ \tau_{xy} \\ \tau_{yy} \end{bmatrix}\boldsymbol{i}_y$$

where β is the pseudocompressibility parameter; p is the pressure; u and v are components of velocity $\boldsymbol{q}$ in the x and y directions, respectively; V is the control volume; ∂V is the boundary of V; and n is the unit outward normal vector to the boundary.

Like the LU-SGS scheme for compressible flows, using flux difference concepts, the contribution of the inviscid flux Jacobians at each cell face is split into positive and negative parts $\boldsymbol{A}^{\pm}$ and $\boldsymbol{B}^{\pm}$, and a splitting discretization can be written as follows:

$$\begin{aligned}&\{\boldsymbol{I} + \alpha(\boldsymbol{A}^{+}_{i,j} - \boldsymbol{A}_{i,j} + \boldsymbol{B}^{+}_{i,j} - \boldsymbol{B}^{-}_{i,j})\}\Delta\boldsymbol{W}_{i,j} \\ &\quad + \alpha(-\boldsymbol{A}^{+}_{i-1,j}\Delta\boldsymbol{W}_{i-1,j} - \boldsymbol{B}^{+}_{i,j-1}\Delta\boldsymbol{W}_{i,j-1}) \\ &\quad + \alpha(\boldsymbol{A}^{-}_{i+1,j}\Delta\boldsymbol{W}_{i+1,j} + \boldsymbol{B}^{-}_{i,j+1}\Delta\boldsymbol{W}_{i,j+1}) = -\alpha\boldsymbol{R}^{n}_{i,j}\end{aligned} \tag{2}$$

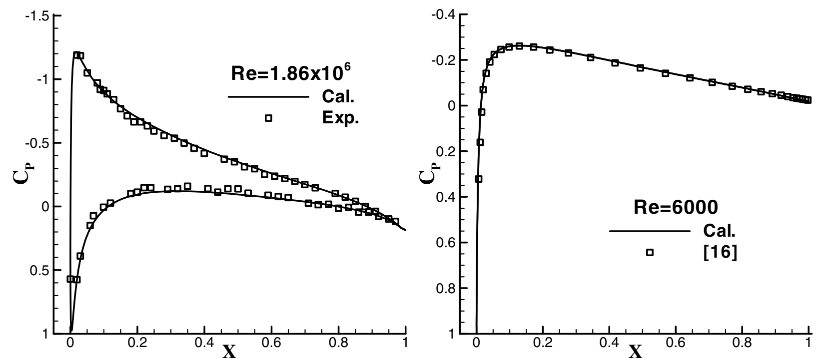
where $\boldsymbol{I}$ is a unit matrix; $\alpha = \Delta t/V_{i,j}$; $\boldsymbol{R} = \boldsymbol{Q}_C + \boldsymbol{Q}_D + \boldsymbol{D}$; $\boldsymbol{Q}_{Ci,j}$ and $\boldsymbol{Q}_{Di,j}$ are, respectively, the net convective and viscous flux out of the cell; and $\boldsymbol{D}$ is the fourth-order artificial dissipative terms.

To ensure a greater diagonal dominance of the LU factors for a well-conditioned implicit algorithm, the splitting proposed by Yoon and Jameson [6] is used in the present work. The Jacobian matrix $\boldsymbol{A} = \boldsymbol{A}^{+} + \boldsymbol{A}^{-}$ is approximated by

$$\boldsymbol{A}^{\pm} = \frac{\boldsymbol{A} \pm r_A\boldsymbol{I}}{2}, \qquad r_A = \chi\max(|\lambda_A|) \tag{3}$$

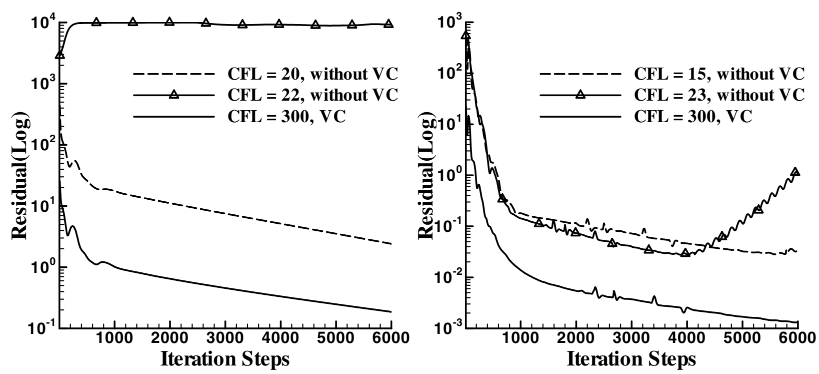
where λ_A is the eigenvalue of inviscid flux Jacobian matrix $\boldsymbol{A}$, and χ is a constant that is greater than or equal to 1. A similar procedure is applied to the Jacobian matrices $\boldsymbol{B}$.

For improving stability and efficiency, the contribution of viscous Jacobians is introduced by an approximate expression [3], and the



a) NACA0012, α = 3.59 deg b) NACA0008, α = 0 deg

Fig. 1 Comparison of computed pressure distribution with the experimental data and reference results.



a) Re = 10^3 b) Re=10^4

Fig. 2 Stability and efficiency of the LU-SGS method at different Reynolds numbers.

Jacobian matrices are modified with viscous approximations as follows:

$$\tilde{\boldsymbol{A}}^{\pm} = \boldsymbol{A}^{\pm} \pm 2\nu|\nabla\xi|^2\boldsymbol{I}, \qquad \tilde{\boldsymbol{B}}^{\pm} = \boldsymbol{B}^{\pm} \pm 2\nu|\nabla\eta|^2\boldsymbol{I} \tag{4}$$

By adding the viscous eigenvalues, the stability of the LU-SGS scheme is further improved and a larger CFL number can be used.

The governing equations are solved with a cell-centered finite volume method on structured grids. The discretized equations with the LU-SGS scheme can be written as follows:

$$(\boldsymbol{L} + \boldsymbol{D})\boldsymbol{D}^{-1}(\boldsymbol{D} + \boldsymbol{U})\Delta\boldsymbol{W}^n = -\alpha\boldsymbol{R}(\boldsymbol{W}^n_{i,j}) \tag{5}$$

where

$$\begin{aligned} \boldsymbol{L} &= -\alpha(\tilde{\boldsymbol{A}}^+_{i-1,j} + \tilde{\boldsymbol{B}}^+_{i,j-1}) \qquad \boldsymbol{U} = \alpha(\tilde{\boldsymbol{A}}^-_{i+1,j} + \tilde{\boldsymbol{B}}^-_{i,j+1}) \\ \boldsymbol{D} &= +\alpha[(r_A + r_B) + 2\nu(|\nabla\xi|^2 + |\nabla\eta|^2)]\boldsymbol{I} \end{aligned} \tag{6}$$

and $\alpha = \Delta t/V_{i,j}$. In the present work, the local time-stepping technique is employed. When the Reynolds number is greater than or equal to 10,000, the Baldwin–Lomax turbulence model is used.

III. Results and Discussion

To illustrate the effects of the present improvement, several incompressible viscous flows are simulated. At first, the implicit code is validated by comparison with the experimental data and others' numerical results. By comparing the performance of the present method with one without viscous correction, the effects of viscous correction on stability and efficiency are also investigated. In all calculations, pseudocompressibility β is equal to 3.

First, to test validation of the method for incompressible viscous problems, the flows over NACA0008 and NACA0012 airfoils are simulated. For these cases, computations are performed on stretched 192×52 C-meshes. In the streamwise direction, 140 mesh cells are located on the airfoil surface. In the normal direction, the nondimensional mesh spacing at the wall is about 1.0×10^{-5}.

These models are designed to study the characteristics of airfoils of incompressible viscous flows, and the pressure distributions are also presented. The first case is for turbulent flow at a high Reynolds number of 1.86×10^6 and an angle of attack of 3.59 deg, as shown in Fig. 1a. The second case is for laminar flow at a low Reynolds number of 6000 and an angle of attack of 0 deg, as shown in Fig. 1b. In Fig. 1, the computed results show good agreement with the experimental data [15] and reference results [16], which indicates that the present scheme is very reliable for solving low-Reynolds-number incompressible flow equations.

To analyze the effect of viscous correction for improving stability and efficiency, computations of incompressible flows over a NACA0012 airfoil were performed at different Reynolds numbers. The Reynolds numbers selected for this analysis are 10^3 and 10^4, with an angle of attack of 0 deg.

The performance of the present scheme is compared with the LU-SGS method without viscous correction in Fig. 2. The figures show the residual histories of divergence of velocity at different Reynolds numbers, with or without viscous correction on the LU-SGS method. In these figures, VC represents viscous correction.

Figure 2 shows that without viscous correction, the maximum CFL number is less than 22 at low Reynolds numbers. With viscous correction, the CFL number can be increased to be as large as 300. It indicates that for low Reynolds numbers, with the contributions of the viscous Jacobians, the CFL number can be increased greatly. As a result, the convergence of computation is improved for low Reynolds numbers. It seems that viscous correction is indispensable for impressible simulations of low Reynolds numbers by considering stability and efficiency.

The results of the investigation demonstrate that with the viscous correction in Jacobian matrix, the present method has the advantage of greatly increasing the CFL numbers at low Reynolds numbers. Consequently, by increasing the CFL numbers, the computational time is reduced and the efficiency is also improved. According to the computational experience, the present method is nearly unconditional stable for low Reynolds number. This is very useful for the incompressible simulation.

IV. Conclusions

The present method was applied to solve a variety of incompressible viscous problems on structured grids. The numerical results obtained in this study demonstrate that the LU-SGS method with viscous correction is more stable and efficient than the method that includes only the inviscid flux Jacobians. The numerical solution can quickly reach the steady state with large CFL numbers, owing to the improving stability, which is very useful for the incompressible simulation of low-Reynolds-number flows.

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